GIS Map of Equilibrium Suction as Controlled by the Soil and Vegetation

FPA PRESENTATION
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TMI Distribution in United States

(After Thornthwaite 1948)
Contributing Factors in TMI

- Precipitation
- Potential Evapotranspiration
- Depth of Available Moisture
- Initial Value for Depth of Moisture
Monthly Moisture Balance

- Rainfall
- Evapo-transpiration
- Storage
- Runoff
TMI Distribution in Texas

(After Lytton et al. 1974)
Witczak’s TMI Model

\[ TMI = 75\left(\frac{P}{PE_y} - 1\right) + 10 \]

where \( P \) = Annual Average Precipitation;
\( PE_y \) = Annual Average Potential Evapotranspiration
Average Annual Precipitation

Precipitation Map in GIS Platform (1981 to 2010)
Average Annual Potential Evapotranspiration Map in GIS Platform (1981 to 2010)

Potential Evapotranspiration Map in GIS Platform (1981 to 2010)
Average Annual TMI

TMI Map in GIS Platform (1981 to 2010)
# Validation of TMI Map

## Collected TMI Values from Original Map

<table>
<thead>
<tr>
<th>Locations</th>
<th>Latitude</th>
<th>Longitude</th>
<th>TMI (original map)</th>
<th>Locations</th>
<th>Latitude</th>
<th>Longitude</th>
<th>TMI (original map)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gallup, New Mexico</td>
<td>35.52</td>
<td>-108.74</td>
<td>-32</td>
<td>Port Arthur, Texas</td>
<td>29.88</td>
<td>-93.93</td>
<td>26.8</td>
</tr>
<tr>
<td>Synder, Texas</td>
<td>32.71</td>
<td>-100.91</td>
<td>-25</td>
<td>Lake Charles, Louisiana</td>
<td>30.22</td>
<td>-93.21</td>
<td>58.2</td>
</tr>
<tr>
<td>Durant, Oklahoma</td>
<td>33.99</td>
<td>-96.39</td>
<td>18.4</td>
<td>Reliance, South Dakota</td>
<td>43.87</td>
<td>-99.60</td>
<td>-12.9</td>
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<tr>
<td>Houston, Texas</td>
<td>29.76</td>
<td>-95.36</td>
<td>14.8</td>
<td>Ellsworth, Kansas</td>
<td>38.73</td>
<td>-98.22</td>
<td>9.1</td>
</tr>
<tr>
<td>San Antonio, Texas</td>
<td>29.42</td>
<td>-98.49</td>
<td>-21.3</td>
<td>Limon, Colorado</td>
<td>39.26</td>
<td>-103.69</td>
<td>-16.8</td>
</tr>
<tr>
<td>El Paso, Texas</td>
<td>31.76</td>
<td>-106.48</td>
<td>-46.5</td>
<td>Price, Utah</td>
<td>39.59</td>
<td>-110.81</td>
<td>-36.4</td>
</tr>
<tr>
<td>Monroe, Louisiana</td>
<td>32.51</td>
<td>-92.11</td>
<td>65.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Validation of TMI Map

Comparison of TMI Values

$R^2 = 0.93$

line of equality
Typical Equilibrium Suction Profile with Depth

- Wet Season
- Equilibrium
- Dry Season
Development of a Modified Equilibrium Suction Model
Modified Equilibrium Suction Model

**Contributing Factors**

- **Soil Properties** [Steady State Diffusivity Equation (Mitchell 1979)]
- **Climatic Factors** [Relationship between TMI and Max Available Moisture Depth (Gay 1994)]
Suction Profile in Unsaturated Soil

Mitchell’s Diffusivity Equation,

\[ u(z) = u_e \pm u_0 \cdot e^{-\sqrt{\frac{\pi n}{\alpha} z}} \]

Suction Profile Between Wet and Dry State
Soil Suction at Surface Ranges Between $pF = 3$ to $pF = 4.5$
Empirically Measured Suctions BCI 2002 to 2008 = 26,000+ Data Points
Total Soil Suction Histogram

2003 TOTAL SOIL SUCTION DATA (4776 OBSERVATIONS)
Depth of Moisture Active Zone, $Z_m$ Varies Between 9.3 and 21 feet Depending on Vegetation
Case Studies of Moisture Active Zone Depth

Around a Root Zone in Louisiana

Around a Root Zone in Texas

(After Lytton et al. 1994)
Fraction of Vegetation Cover Map

From NOAA Climatic Data Records
Typical Suction Vs Water Content Curve

(After Sahin 2014)
Diffusivity Coefficient

\[ \alpha = \frac{k_{\text{sat}}}{\frac{\partial \theta_w}{\partial h}} \]

where \( k_{\text{sat}} = \) Saturated Permeability;
\( \partial \theta_w / \partial h = \) Slope of SWCC Curve
From Fredlund-Xing Equation

\[ \theta_w = C(h) \times \left[ \frac{\theta_{sat}}{h} \right] \left\{ \ln[\text{e} + \left( \frac{h}{a_f} \right)^{b_f}] \right\}^{c_f} \]

\[ C(h) = 1 - \frac{\ln(1 + \frac{h}{h_r})}{\ln[1 + \left( \frac{1.021 \times 10^7}{h_r} \right)]} \]

\[ a_f, b_f, c_f \text{ and } h_r \text{ are Fitting Parameters} \]
Prediction Model for SWCC Fitting Parameters

**Plastic Soil**

**Non-Plastic Soil**

(After Saha et al. 2018)
Available Annual Moisture Depth, $d_{am}$

$$d_{am} = \int_{0}^{z_m} [\theta_{wet}(z) - \theta_{dry}(z)]$$

$$= \text{Area } ABC + \text{Area } BCDE$$

$$\frac{\theta_{dry}' - \theta_{dry}}{\sqrt{\frac{n\pi}{\alpha}}} \left( 1 - e^{-\frac{\sqrt{n\pi} z_m}{\alpha}} \right) + (\theta_{wet}' - \theta_{dry}') z_m$$

Maximum Moisture Stored Between Dry and Wet Profile
Mean Annual Moisture Depth Vs TMI

(After Gay 1994)
Relationship Between TMI and Moisture Depth

Using Juarez-Badillo’s approach

\[
\gamma \frac{dT}{T} = \frac{dF(d)}{F(d)}
\]

\[
\gamma \ln \left[ \frac{T}{T_1} \right] = \ln \left[ \frac{1}{d_{am} - d_m} - \frac{1}{1} \right] - \ln \left[ \frac{1}{d_{am} - d_1} - \frac{1}{d_{am}} \right]
\]
Mean Annual Moisture Depth

Using Juarez-Badillo’s approach

\[ d_m = \frac{d_{am}}{[1 + \frac{d_{am} - d_1}{d_1(T/T_1)^\gamma}]} \]

where \( T = TMI + 65 \);
\( d_m = \text{Mean Annual Moisture Depth} \);
\( \gamma, T_1 \text{ and } d_1 \) are Regression coefficient
Validation of Equilibrium Suction Model
## Significant Parameters of Equilibrium Suction

<table>
<thead>
<tr>
<th>Variables</th>
<th>Degree of freedom</th>
<th>Parameter estimate</th>
<th>Standard error</th>
<th>t Ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1</td>
<td>1.267</td>
<td>0.406</td>
<td>3.118</td>
<td>0.0021</td>
</tr>
<tr>
<td>TMI</td>
<td>1</td>
<td>-0.00114</td>
<td>0.000593</td>
<td>-1.9306</td>
<td>0.0451</td>
</tr>
<tr>
<td>PI</td>
<td>1</td>
<td>0.0297</td>
<td>0.00346</td>
<td>8.5991</td>
<td>4.41E-15</td>
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<tr>
<td>$u_{dry}$</td>
<td>1</td>
<td>0.5153</td>
<td>0.0647</td>
<td>7.9623</td>
<td>2.054E-13</td>
</tr>
<tr>
<td>$Z_m$</td>
<td>1</td>
<td>0.00046</td>
<td>0.000318</td>
<td>1.444</td>
<td>0.1503</td>
</tr>
<tr>
<td>$F_r$</td>
<td>1</td>
<td>-0.1153</td>
<td>0.1859</td>
<td>-0.6203</td>
<td>0.5358</td>
</tr>
<tr>
<td>$\sqrt{1/\alpha}$</td>
<td>1</td>
<td>-0.0042</td>
<td>0.00084</td>
<td>-4.9505</td>
<td>1.731E-06</td>
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<tr>
<td>$\theta_{\alpha}$</td>
<td>1</td>
<td>-0.4327</td>
<td>0.3182</td>
<td>-1.36</td>
<td>0.175</td>
</tr>
<tr>
<td>$a_f$ (pF)</td>
<td>1</td>
<td>-0.00174</td>
<td>0.0265</td>
<td>-0.0656</td>
<td>0.9477</td>
</tr>
<tr>
<td>$b_f$</td>
<td>1</td>
<td>0.00349</td>
<td>0.00617</td>
<td>0.5665</td>
<td>0.5717</td>
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<tr>
<td>$c_f$</td>
<td>1</td>
<td>-0.1758</td>
<td>0.1183</td>
<td>-1.4859</td>
<td>0.1391</td>
</tr>
</tbody>
</table>
Equilibrium Suction Vs TMI

AASHTO soil type:
A-1

AASHTO soil type:
A-2
Equilibrium Suction Vs TMI

AASHTO soil type: A-3

AASHTO soil type: A-4

St. Deviation, $\sigma = 0.09$
y = 3.370e$^{-3E-04x}$

St. Deviation, $\sigma = 0.17$
y = 3.621e$^{-3E-04x}$
Equilibrium Suction Vs TMI

AASHTO soil type: A-6

AASHTO soil type: A-7-6
Equilibrium Suction Vs Vegetation Cover

St. Deviation, $\sigma = 0.18$

$$y = -0.093 \ln(x) + 3.3652$$

St. Deviation, $\sigma = 0.23$

$$y = -0.09 \ln(x) + 3.3967$$

TMI: $>40$

TMI: 40 to 10
Equilibrium Suction Vs Vegetation Cover

St. Deviation, $\sigma = 0.18$
$y = -0.088 \ln(x) + 3.4025$

St. Deviation, $\sigma = 0.21$
$y = -0.084 \ln(x) + 3.4539$

TMI: 10 to -5  
TMI: -5 to -25
Equilibrium Suction Vs Vegetation Cover

St. Deviation, $\sigma = 0.23$

$y = -0.057 \ln(x) + 3.5358$

St. Deviation, $\sigma = 0.24$

$y = -0.054 \ln(x) + 3.49$

TMI: -25 to -40

TMI: < -40
## Depth of Constant Suction Based on TMI Intervals

### Climatic Divisions in Australia in AS2870-2011

<table>
<thead>
<tr>
<th>Climatic Divisions in Australia</th>
<th>TMI</th>
<th>Depth to Constant Suction (m)</th>
<th>Equilibrium Suction (pF)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AS2870-1996</td>
<td>AS2870-2011</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>Alpine/coastal</td>
<td>TMI &gt; 40</td>
<td>TMI &gt; 10</td>
</tr>
<tr>
<td></td>
<td>10 &lt; TMI &lt; 40</td>
<td>-5 &lt; TMI &lt; 10</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>Wet temperate</td>
<td>-5 &lt; TMI &lt; 10</td>
<td>-15 &lt; TMI &lt; -5</td>
</tr>
<tr>
<td>III</td>
<td>Temperate</td>
<td>-25 &lt; TMI &lt; -5</td>
<td>-25 &lt; TMI &lt; -15</td>
</tr>
<tr>
<td>IV</td>
<td>Dry temperate</td>
<td>TMI &lt; -25</td>
<td>-40 &lt; TMI &lt; -25</td>
</tr>
<tr>
<td>V</td>
<td>Semi-arid</td>
<td>N/A</td>
<td>TMI &lt; -40</td>
</tr>
<tr>
<td>VI</td>
<td>Arid*</td>
<td></td>
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</tr>
</tbody>
</table>

Climatic divisions in Australia in AS2870-2011
Thank You !!!!!
Development of a Modified Equilibrium Suction Model for Subgrade Layers

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