Stability of Slopes and Earth Structures Constructed of High Plasticity Clays

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Outline

• The Problem
• Model for Slope Failures
• Soil Suction
• Case Histories
• Cracking
• Moisture Diffusion
• Suction-Soil Strength Relationship
• Other Earth Structures
Issues
(TxDOT Project 2000-2002)

- Mechanism of Slope Failures
- Evaluation of Relevant Soil Properties
- Analytical Models of Strength Degradation
- Expected Strength Loss over Time and Location
RUNOFF WATER

WATER SOAKS INTO SOIL

SUCTION RANGE BETWEEN CRACKS

WET LIMIT

DRY LIMIT

4.0

2.0

2.0

2.5

pF
MECHANISM OF SHALLOW SLOPE FAILURE

FS = h_m f \Theta \sin \phi' / \gamma H \cos \beta \sin \beta (1+\sin \phi')

\( \theta \): VOLUMETRIC WATER CONTENT
\( h_m \): MATRIC SUCTION
\( \phi' \): EFFECTIVE FRICTION ANGLE

Matric Suction
Soil Suction

\[ h_t = h_m + \pi \]

- **Total**: Controls Flow Through Soil
- **Matric**: Controls Soil Strength, Volume Change
- **Osmotic**: \( f \) (pore salts)
### pF SUCTION SCALE

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\[
pF = \log_{10} \{h \text{ (cm)}\}
\]
The graph illustrates the relationship between matric suction (psf) and shear strength (psf) for different values of pF and phi. The equation $	au_f = h_m f \theta \tan \phi'$ is shown, where $\tau_f$ is the shear strength, $h_m$ is the height, $f$ is the factor, $\theta$ is the angle, and $\phi'$ is the effective angle of friction. The graph includes lines for pF values of 4.0, 3.5, 3.0, and 2.5, corresponding to phi values of 30°, 30°, 30°, and 25°, respectively. The x-axis represents matric suction (psf), and the y-axis represents shear strength (psf).
Case Histories

Data Source: Kayyal & Wright, 1991

- Paris Clays - 16 failures
- Beaumont Clays - 18 failures

TAMU Interpretation: Aubeny & Lytton

- Infinite Slope Analysis
- Soil Strength Derived from Suction
- $c' = 0$

$$FS = \frac{h_m f \Theta \sin \phi' / \gamma H \cos \beta \sin \beta}{(1+\sin \phi')}$$
Paris Clays

- 16 failed slopes
- LL = 80, PL = 22
- Depth of failure mass: 2-10 ft
- Slope: 2.3-3.0 Horizontal to 1 Vertical

- Back-calculated *matric* suction at failure:
  \[ pF \ 2.23 \pm 0.18 \]
Beaumont Clays

- 18 failed slopes
- LL = 73, PL = 21
- Depth of failure mass: 2.4-5 ft
- Slope: 2.5-3.1 Horizontal to 1 Vertical

- Back-calculated *matric* suction at failure:
  \[ \text{pF } 2.05 \pm 0.14 \]
Chronology of Slope Failure

- Post-Construction: pF $\sim 4$, high strength

- Surface Cracks
  $f$ (root depth, climate, soil)

- Moisture Infiltration into Soil
  $f$ (crack depth, climate, soil $\alpha$)

- Suction Reduction / Strength Loss
  $f$ (suction, time, friction $\phi'$)
Moisture Diffusion through Unsaturated Soil

- Mitchell’s Formulation
- Diffusion Coefficient, $\alpha$
- Generalized Formulation
- Analytical/Numerical Solutions
Permeability versus Suction

\[ k = k_0 \left(\frac{h_0}{h}\right)^n \]

- \( k_0 \): saturated k (pF=2)
- \( h_0 \): reference h (pF=2)
- \( n \): material constant
Flow Through Partly Saturated Soil

Mitchell Assumption, n=1

\[ k = k_0 \left( \frac{h_o}{h_t} \right)^n \]

\[ u(pF) = \log_{10} h_t (cm) \]

Darcy’s Law:

\[ V = -k \frac{dh_t}{dx} = -k_0 h_0 \frac{dh_t}{dx} = -k_0 h_0 \frac{d}{dx} \left( h_t / h_t \right) = -k_0 h_0 \frac{d \log_{10} h_t}{dx} = -k_0 h_0 \frac{d \log_{10} h_t}{dx} = -k_0 h_0 \frac{d \log_{10} h_t}{dx} \]

\[ V = -\left[ \frac{k_0 h_0}{0.434} \right] \frac{du}{dx} = p \frac{du}{dx} \]

Equation Linear in \( u(pF) \)

=> flow nets apply

=> analytical solutions apply
Suction versus Water Content

c = dw/du

Field Capacity
Plastic Limit
Wilting Point
Tensile Strength of Water
Air Dry
Oven Dry

Water Content, w

0.88 n

n, Porosity
**Fluid Flow Through Soil**  
(after Mitchell, 1980)

### Unsaturated

\[ \alpha \cdot \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \]

- \( \alpha = \frac{p}{\gamma_d c} \)
- Diffusion Coefficient

- \( u = \log h_t \)

### Saturated (Terzaghi)

- \( c_v \cdot \frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial t} \)

- \( c_v = \frac{k}{m_v \gamma_w} \)
- Consolidation Coefficient

- \( h = \) total head
Advantages of Mitchell’s Formulation

1. Closed-form solutions possible

2. Graphical solutions (flow nets) possible

3. Simple FEM analysis for complex geometry

4. Straight-forward evaluation of material properties (α)
Shelby Tube Sample
Boundary suction function of relative humidity in air

- Drill Holes
- Insert Psychrometers
- Measure suction over time

Evaporation from open end

Sealed End Cap

Shelby Tube

OPEN END – DIFFUSION TEST
Shelby Tube and Psychrometers
Assembled Diffusion Test
Bulb Humidity Equipment
Coordinate, $x/L$

Sealed end

Open end

$T = \alpha t/L^2$

$\left( \frac{u_e}{u_a} \right)$

$T = 0.01$

$0.20 0.15 0.10 0.05$

$0.30 0.40 0.50 0.75$

$0.6 0.8 1.0$

$0 0.2 0.4 0.6 0.8 1.0$
Test 2 - Psychrometer 6

Best Fit $\alpha = 0.094 \text{ cm}^2/\text{sec}$
Permeability versus Suction

\[ k = k_0 \left( \frac{h_0}{h} \right)^n \]

- \( k_0 = \) saturated k (pF=2)
- \( h_0 = \) reference h (pF=2)
- \( n = \) material constant

Permeability versus Suction
Generalization of Mitchell’s Formulation
(n not equal 1)

\[ \dot{\alpha} \left( \frac{\partial^2 \varnothing}{\partial x^2} \right) = \frac{\partial \varnothing}{\partial t} \]

\[ \varnothing = \begin{cases} \log_e h & n = 1 \\ (1 - n)h^{1-n} & n \neq 1 \end{cases} \]

- Equation still linear
- 2 Material Parameters, \( \alpha' \) and \( n \)
Significance of $\alpha$

Variation in Suction at Surface

Depth of influence increases with increasing $\alpha$, $T$

Analytical Solution

$$u = u_e + u_0 \exp \left( -\sqrt{\frac{\partial}{\alpha T}} x \right) \cos \left( 2\partial \left( t - \sqrt{\frac{\partial}{\alpha T}} x \right) \right)$$
RUNOFF WATER

WATER SOAKS INTO SOIL

SUCTION RANGE BETWEEN CRACKS

DRIp LIMIT

WET LIMIT

2.0

2.5

4.0

pF

2.0
Suction-Strength Relationship

• Total Suction: $h(t)$ from moisture diffusion analysis

• Matric Suction: $h_m = h - \pi$
  \[
  \pi = \text{Osmotic suction} = f \text{ (pore water salts)}
  \]

• ‘Cohesive’ Strength: $c_{\text{app}} = \theta f h_m \sin \phi'/(1- \sin \phi')$
  \[
  \theta = \text{volumetric water content}
  \]
  \[
  f \text{ varies from } 1 \text{ to } 1/\theta \text{ as full saturation approached}
  \]
  \[
  \phi' = \text{friction angle}
  \]
Other Earth Structures
Definition Sketch

- 5 ft cracks w/moisture entering
- Strength profile through mid-height

Dimensions:
- Width: 40 ft
- Height: 20 ft
Strength Profile vs Time (mid-height of wall)

\[ \alpha = 10^{-3} \text{ cm}^2/\text{s} \]
\[ \alpha = 10^{-4} \text{ cm}^2/\text{s} \]

Horizontal Distance From Left Wall (ft)

Strength (psf)

- T=0.1 1.2 years
- T=0.5 6 years
- T=1.0 12 years
- T=5.0 60 years
- T=10.0 120 years
Model for Cracking
SHALLOW SLOPE FAILURE

DURING DRY PERIODS ROOTS EXTRACT WATER FROM THE SOIL AND CAUSE SHRINKAGE CRACKS
Crack Spacing Gets Larger with Depth
SOURCE: MICHAEL KNIGHT
PH. D. DISSERTATION, GEOLOGY
UNIVERSITY OF MELBOURNE (AUSTRALIA)
1972
WET SUCTION LIMIT
2.5

DRY SUCTION-WILTING POINT
4.5

ROOT ZONE

TRANSIENT SUCTION

WETTING SUCTION ENVELOPE

DRYING SUCTION ENVELOPE

$U_e$

EQUILIBRIUM SUCTION
(TYPICALLY 3.0 – 4.2 IN TEXAS)
\[ d_c = d_r + z_t \]
How deep do the cracks go?
How does cementation affect the crack depth?
How do cemented soils behave in different climates?
alpha=0.001

T (week)

depth (cm)

ue=3, uo=1.5
ue=3.5, uo=1.0
ue=4, uo=0.5
\( \alpha = 0.00001 \)

\( T(\text{week}) \)

\( \text{ue}=3.0, \ uo=1.5 \)

\( \text{ue}=3.5, \ uo=1.5 \)

\( \text{ue}=4.0, \ uo=0.5 \)
Conclusions

- Decrease in suction related to slope failures
- Moisture diffusion controls rate of strength loss
- Critical parameters:
  - Depth of root zone
  - Crack depth
  - Diffusion coefficient, $\alpha$
  - Osmotic suction, $\pi$
  - Friction angle, $\phi'$
Evaluation of Parameters

• Depth of root zone
  field reconnaissance
• Crack depth
  field reconnaissance, predictive model
• Diffusion coefficient, $\alpha$
  laboratory measurement, correlation to index properties
• Osmotic suction, $\pi$
  laboratory measurement, regional databases
• Friction angle, $\phi'$
  laboratory measurement, correlation index properties
EDGE MOISTURE DISTANCE

\[ \alpha (cm^2/sec), \text{Unsaturated Diffusivity Coefficient} \]

\[ 0.0E+0 \quad 1.0E-3 \quad 2.0E-3 \quad 3.0E-3 \quad 4.0E-3 \quad 5.0E-3 \quad 6.0E-3 \quad 7.0E-3 \quad 8.0E-3 \]

\[ 0.25 \quad 0.20 \quad 0.15 \quad 0.10 \quad 0.05 \quad 0.00 \]

PLASTICITY INDEX (PI, %)

CENTER LIFT

EDGE LIFT

VOLUME COMPRESSION CHANGE PRESSION COEF
Fig. 11.27 Empirical correlation between $\phi'$ and PI from triaxial compression tests on normally consolidated undisturbed clays (after U.S. Navy, 1971, and Ladd, et al., 1977).